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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SIXTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: EE306

Course Name: POWER SYSTEM ANALYSIS (EE)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 A 120 MVA, 19.5 kV generator has X = 1.5 percent and is connected to a transmission line by a star-delta transformer rated 150 MVA, 230/18 kV with X = 0.1 percent. If the base to be used in the calculations is 100MVA, 230kV for the transmission line, find the per unit values to be used for the transformer and generator reactances. (5)

2 A single line to ground fault occurs at the terminals of a 30 MVA, 11 kV generator. The positive, negative and zero sequence impedances in pu are j0.2, j0.2 and j0.05 respectively. Find the line currents under faulted conditions. Assume that the generator is solidly grounded. (5)

3 What are the main functions of load frequency controller in power system? (5)

4 Classify the various types of buses in a power system for load flow studies. (5)

5 The fuel cost functions for three thermal plants in Rs/hr. are given by, (5)

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2$$
$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$
$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

Find the power generated by each plant if the total demand is 800MW.

6 What do you mean by penalty factor as referred to economic operation of power system? (5)

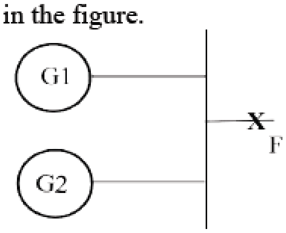
7 What are the factors affecting transient stability in power system? (5)

8 What is swing equation? Derive the expression for swing equation for a synchronous machine connected to an infinite bus. (5)

PART B

Answer any two full questions, each carries10 marks.

9 (a) Two 11 kV, 3-phase generators rated 10 MVA, 25% and 20 MVA, 40% operate in parallel. Calculate the short circuit kVA if a three phase short circuit occurs on the feeder at point 'F' as shown in the figure. (6)

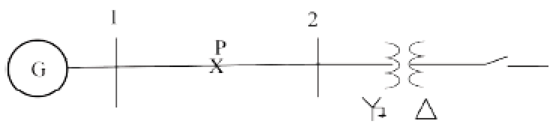


Calculate the reactance value of the feeder reactor to be included so that the short circuit kVA is reduced by 50%.

(b) Find the expression for three phase power in terms of symmetrical components. (4)

10 A synchronous generator and motor are rated 30MVA, 13.2kV and both have sub-transient reactances of 20%. The line connecting them has a reactance of 10% on the base of the machine ratings. The motor is drawing 20,000kW at 0.85 power factor lagging at a terminal voltage of 12.8 kV when a symmetrical three phase fault occurs at the motor terminals. Find the sub-transient current in the generator, motor and fault. (10)

11 (a) A three phase generator is connected to a star-delta transformer as shown in the figure. (6)



The reactance values referred to a common base are :

	Z_1	Z_2	Z_0
Alternator	j0.1	j0.1	j0.05
Transformer	j0.05	j0.05	j0.05
Transmission line	j0.4	j0.4	j0.8

Determine the fault current when a double line to ground fault occurs at point 'P' at the mid-point of the line if the alternator neutral is grounded. Assume that the generator is not loaded.

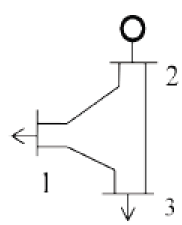
(b) Obtain the symmetrical components of the following set of unbalanced currents (4)
$I_a = 1.6\angle 250^0$, $I_b = 1.0\angle 180^0$, $I_c = 0.9\angle 132^0$.

PART C

Answer any two full questions, each carries 10 marks.

12 Explain the computational procedure for load flow solution using fast decoupled load flow method. (10)

13 Figure shows a three bus power system. The impedance of each line is (0.026 + j0.11) pu. (10)



The bus details are given in the table below

Bus	P_G(pu)	Q_G(pu)	P_L(pu)	Q_L(pu)	|Vi|(pu)	Angle	Remarks
1	-	-	1.0	0.5	1.03	0^0	Slack bus
2	1.5	-	0	0	1.03	-	PV bus
3	0	0	1.2	0.5	-	-	PQ bus

Assuming a flat voltage start, find the voltages and bus angles at the buses at the end of the first iteration using Gauss-Siedel method.

14 A two area system connected by a tie line has the following parameters on a 1000 MVA common base. (10)

Area	1	2
Speed regulation	0.05	0.0625
Frequency sensitive load co-efficient	0.6	0.9
Inertia constant	5	4
Governor time constant	0.2	0.3
Turbine time constant	0.5	0.6

The units are operating in parallel at a nominal frequency of 60 Hz. The synchronizing power co-efficient is given as 2.0 pu. If the load in area1 increases by 187.5 MW, determine the new steady state frequency and the change in tie-line flow.

PART D

Answer any two full questions, each carries 10 marks.

15 Prove that the maximum permissible sudden increase in load is 72.5% of the steady state limit if the machine is initially at no load. (10)

16 (a) Explain Equal Area criterion and state the assumptions made. (5)

(b) Derive the expression for transmission losses as a function of power generation. (5)

17 (a) What is unit commitment problem? What are the constraints and the solution techniques for unit commitment problem involving thermal plants? (5)

(b) Find the energy stored in the rotor of a three phase, 50 Hz. 250 MVA turbo alternator with H=7.5 MJ/MVA. Determine the value of the inertia constant M. The generator is initially supplying a steady power of 150 MW. If the mechanical power input to the turbine is suddenly decreased to 100 MW, evaluate the initial acceleration of the rotor neglecting all losses. Assume 6 poles. Also find the rotor speed after 10 cycles. (5)

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1 A 120 MVA, 19.5 kV generator has X = 1.5 percent and is connected to a transmission line by a star-delta transformer rated 150 MVA, 230/18 kV with X = 0.1 percent. If the base to be used in the calculations is 100MVA, 230kV for the transmission line, find the per unit values to be used for the transformer and generator reactances.

$KV_{b,new} = 230$ kV ; $MVA_{b,new} = 100$ MVA

$X_T = 0.1\%$

$X_g = 1.5\%$

$$X_{G,p.u\,new} = 0.001 \times \cancel{19.5} \left(\frac{230}{230}\right)^2 \times \frac{100}{150}$$

$$= \underline{0.00067 \text{ p.u}} \quad \text{or} \quad \underline{6.67 \times 10^{-4} \text{ p.u}}$$

generator connected to LT side of Tfr.

Base kV referred to LT side of Transformer = Base kV on HT side × LT vtg rating / HT vtg rating

$$= 230 \times \frac{18}{230} = 18 \text{ kV}.$$

$$X_{G,new} = 0.015 \times \left(\frac{19.5}{18}\right)^2 \times \left(\frac{100}{120}\right) = \underline{0.0147 \text{ p.u}}.$$

2 A single line to ground fault occurs at the terminals of a 30 MVA, 11 kV generator. The positive, negative and zero sequence impedances in pu are j0.2, j0.2 and j0.05 respectively. Find the line currents under faulted conditions. Assume that the generator is solidly grounded.

(2) For a single line to ground fault

$I_{a1} = I_{a2} = I_{a0}$ and $I_f = I_a = 3I_{a1}$

$$I_{a1} = \frac{1\angle 0}{j0.2 + j0.2 + j0.05} = \underline{-2.22j} = \underline{2.22\angle -90^\circ} \text{ p.u}.$$

fault current, $I_f = I_a = 3I_{a1} = -6.67j$

$$= \underline{6.67 \angle -90} \text{ p.u}.$$

Base current, $I_b = \frac{KVA_b}{\sqrt{3}\, KV_b} = \frac{30 \times 1000}{\sqrt{3} \times 11} = 1574.59\ A$.

Actual value of fault current = p.u value of fault current × Base current.

$= 6.67 \angle -90 \times 1574.59$

$= 10502.53 \angle -90^\circ = -10502.51j\ A$.

$= 10.502 \angle -90^\circ\ kA$

5 The fusel cost functions for three plants in Rs/hr. are given by,

$C_1 = 500+5.3P_1+0.004P_1^2$

$C_2 = 500+5.3P_2+0.004P_2^2$

$C_3 = 500+5.3P_3+0.004P_3^2$

$\frac{dC_1}{dP_1} = 5.3 + 0.008 P_1$

$\frac{dC_2}{dP_2} = 5.5 + 0.012 P_2$

$P_D = P_1 + P_2 + P_3 = 800 \rightarrow (1)$

$\frac{dC_3}{dP_3} = 5.8 + 0.018 P_3$

$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3}$.

$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} \Rightarrow 5.3 + 0.008 P_1 = 5.5 + 0.012 P_2$.

$P_1 = 687.5 - 662.5 + 1.5 P_2$.

$P_1 = 25 + 1.5 P_2 \rightarrow (2)$.

$$① \rightarrow \quad 800 = 25 + 1.5\,P_2 + P_2 + P_3$$

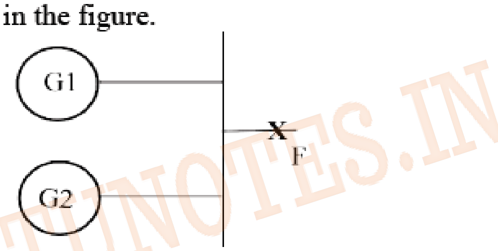
$$775 = 2.5\,P_2 + P_3 \rightarrow (3)$$

$$\frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} \Rightarrow 5.5 + 0.012\,P_2 = 5.8 + 0.018\,P_3$$

$$0.012\,P_2 - 0.018\,P_3 = 0.3 \rightarrow (4)$$

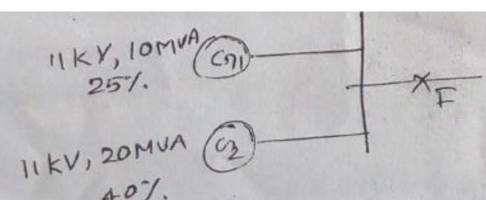
$P_2 = \underline{250}$ MW, $P_3 = \underline{150}$ MW ; $P_1 = 800 - (250+150) = \underline{400}$ MW

9 (a) Two 11 kV, 3-phase generators rated 10 MVA, 25% and 20 MVA, 40% operate in parallel. Calculate the short circuit kVA if a three phase short circuit occurs on the feeder at point 'F' as shown in the figure. (6)



Calculate the reactance value of the feeder reactor to be included so that the short circuit kVA is reduced by 50%.

(a)



$kV_{L,new} = 11$ kV , $MVA_{B,new} = 20$ MVA

$$X_{T1,\,p.u} = 0.25 \times \left(\frac{11}{11}\right)^2 \times \frac{20}{10} = \underline{0.5\ p.u}$$

$(2/9)$

$X_{T2,\,p.u} = \underline{0.4\ p.u}$ $\quad Z_{Th} = \underline{0.222}$ p.u $(0.5 \,\|\, 0.4)$

$$SCC \Rightarrow \frac{S_b}{Z_{Th}} = \frac{20}{0.22(2/9)} \text{ MVA} = \frac{20\times 9}{2} \Rightarrow 90 \text{ MVA}$$
$$\Rightarrow 90\times10^3 \text{ KVA}$$

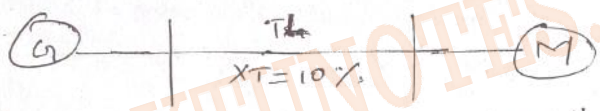
* If short circuit KVA reduced by 50%.

SCC new = 45,000 KVA

$$\therefore Z_{Th} = \frac{20\times10^3}{45\times10^3} = \frac{4}{9} = 0.44 \text{ p.u}$$ (twice)

(176 ← page)

10 A synchronous generator and motor are rated 30MVA, 13.2kV and both have sub-transient reactances of 20%. The line connecting them has a reactance of 10% on the base of the machine ratings. The motor is drawing 20,000kW at 0.85 power factor lagging at a terminal voltage of 12.8 kV when a symmetrical three phase fault occurs at the motor terminals. Find the sub-transient current in the generator, motor and fault.



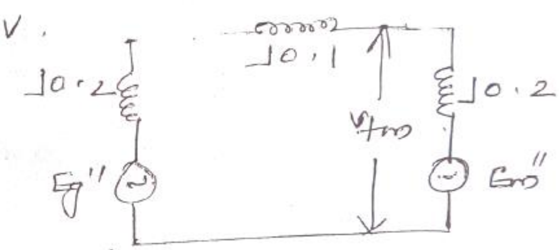
$x_d'' = 20\%$

30 MVA, 13.2 kV

$x_d'' = 20\%$, 12.8 kV

20 MW, 0.85 pf lag.

$MVA_b = 30$ MVA, $kV_b = 13.2$ kV.



Base Current, $I_b = \frac{kVA_b}{\sqrt{3}\, kV_b}$

$$= \frac{30\times10^3}{\sqrt{3}\times 13.2} = 1312.16 \text{ A}$$

Actual value of prefault voltage at fault point, $V_{tm} = 12.8\ kV$

p.u value of prefault vtg at fault point, $V_{tm} = \frac{Actual}{base} = \frac{12.8}{13.2}$

$$V_{tm} = \underline{\underline{0.9697}}\ p.u$$

Actual value of real power of the load, $P_m = 20\,MW, 0.85$ lag.

$$P_m\ (p.u) = \frac{20}{30} = \underline{\underline{0.6667}}\ p.u. \qquad P = VI\cos\phi.$$

p.u value of magnitude of load current, $|I| = \frac{P_m}{V_{tm}\cos\phi}$

$$= \frac{0.6667}{0.9697 \times 0.85} = \underline{\underline{0.8089}}\ p.u.$$

KVL method

prefault condition

$$E_g'' = j0.2 I_L + j0.1 I_L + V_{tm}$$

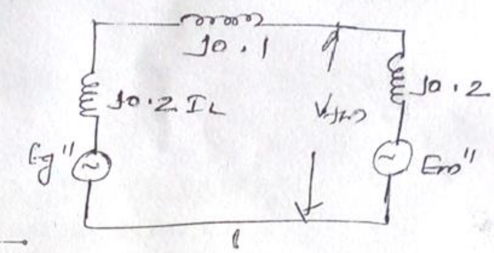
$V_{tm} = \underline{\underline{0.9697}}\angle 0^\circ\ p.u$

$I_L = 0.8089 \angle \cos^{-1} 0.85$

$= \underline{\underline{0.8089}}\angle 31.79\ p.u$

$E_g'' = j0.3\, I_L + V_{tm}$

$= j0.3 \times 0.8089 \angle 31.79 + 0.9697 \angle 0°$



$E_g'' = 0.8418 + j\,0.2063$

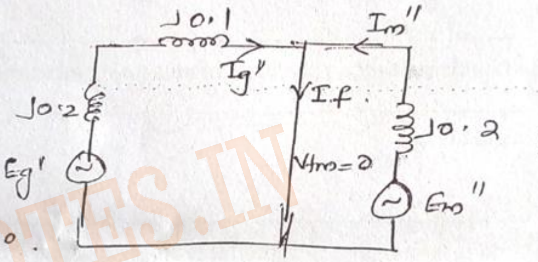
$= 0.8667 \angle 13.77°$

$E_m'' + j0.2 I_L = V_{tm} \Rightarrow E_m'' = 0.9697 \angle 0° - 0.2\angle 90 \times 0.8089 \angle 31.79$

$= 1.055 - 0.1375j$

$= 1.0638 \angle -7.43°$ p.u

Fault condition.



$j0.2 I_g'' + j0.1 I_g'' = E_g''$

$j0.3 I_g'' = E_g''$

$I_g'' = \dfrac{0.8667 \angle 13.77°}{0.3 \angle 90°} = 0.6876 - 2.8059j$

$I_g'' = 2.889 \angle -76.23°$

$j0.2 I_m'' = E_m'' \Rightarrow I_m'' = \dfrac{1.0638 \angle -7.43}{0.2 \angle 90°}$

$= -0.6878 - 5.2743j = 5.319 \angle -97.43°$

Current in the fault during subtransient state $\Big\}$ $I_f'' = I_g'' + I_m''$

$= -1.71 \times 10^{-4} - 8.08j$

$= 8.08 \angle -90.001$ p.u

Actual values

$I_g'' = 2.889 \angle -76.23 \times 1312.16 = 3790.83 \angle -76.23°$ A

$I_m'' = 5.319 \angle -97.43 \times 1312.16 = 6979.38 \angle -97.43°$ A

$I_f'' = 8.08 \angle -90 \times 1312.16 = 10602.2528 \angle -90°$ A

11 (b) Obtain the symmetrical components of the following set of unbalanced currents
$I_a = 1.6\angle 250^0$, $I_b = 1.0\angle 180^0$, $I_c = 0.9\angle 132^0$.

$I_a = 1.6\angle 250°$, $I_b = 1\angle 180°$, $I_c = 0.9\angle 132°$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$(1\angle -60°)$

$a I_b = 1\angle 120° \times 1\angle 180° = 1\angle 300° = 0.5 - 0.866j$

$(1\angle 60°)$

$a^2 I_b = 1\angle 240° \times 1\angle 180° = 1\angle 420° = 0.5 + j0.866$

$0.9\angle -108°$

$a I_c = 1\angle 120° \times 0.9\angle 132° = 0.9\angle 252° = -0.278 - 0.8559j$

$a^2 I_c = 1\angle 240° \times 0.9\angle 132° = 0.9\angle 372° = 0.8803 + 0.187j$

$(0.9\angle 12°)$

$I_{a0} = \frac{1}{3}[I_a + I_b + I_c] = -0.7165 - 0.2782j$

$= 0.7686\angle -158.77$

$I_{a1} = \frac{1}{3}[I_a + aI_b + a^2 I_c]$

$= 0.2777 - 0.7275j = 0.7787\angle -69.11°$

$I_{a2} = \frac{1}{3}[I_a + a^2 I_b + aI_c] = -0.1084 - 0.4978j$

$= 0.5095\angle -102.29°$

Zero sequence components are

$I_{a0} = I_{b0} = I_{c0} = 0.7686\angle -158.77°$

positive sequence

$I_{b1} = a^2 I_{a1} = -0.7683 + 0.1232j = 0.7781\angle 170.89°$

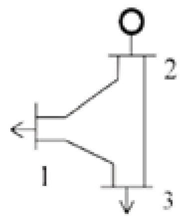
$I_{c1} = a I_{a1} = -0.4912 + 0.6042j = 0.7787\angle 50.89°$

Negative Sequence -

$I_{b2} = aI_{a2} = 0.48535 + j\,0.1549 = 0.5095 \angle 17.71$

$I_{c2} = a^2 I_{a2} = -0.3769 + j\,0.3428 = 0.5095 \angle 137.71^\circ$

13. Figure shows a three bus power system. The impedance of each line is (0.026+j0.11) pu.



The bus details are given in the table below

Bus	P_G(pu)	Q_G(pu)	P_L(pu)	Q_L(pu)	Vi(pu)	Angle	Remarks
1	-	-	1.0	.5	1.03	0^0	Slack bus
2	1.5	-	0	0	1.03	-	PV bus
3	0	0	1.2	0.5	-	-	PQ bus

Assuming a flat voltage start, find the voltages and bus angles at the buses at the end of the first iteration using Gauss-Siedel method.

$$z_{12} = z_{13} = z_{23} = 0.026 + j0.11$$

$$y_{12} = y_{13} = y_{23} = \underline{\underline{2.035 - 8.609j}}$$

$$Y_{bus} = \begin{bmatrix} 4.07 - j17.219 & -2.035 + j8.609 & -2.035 + j8.609 \\ -2.035 + j8.609 & 4.07 - j17.219 & -2.035 + j8.609 \\ -2.035 + j8.609 & -2.035 + j8.609 & 4.07 - j17.219 \end{bmatrix}$$

$$V_1^0 = V_1^1 = \underline{\underline{1.03 + j0}}$$

$$V_2^0 = 1.03 + j0 \qquad \Rightarrow P_2 = 1.5$$

$$V_3^0 = 1 + j0 \qquad \Rightarrow P_3 = 1.2, \; Q_3 = 0.5$$

$$Q_{p,cal}^{k+1} = -1 \times Im\left[(V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right]$$

$$Q_{2,cal}^1 = -1 \times Im\left[(V_2^0)^* \left[\sum_{q=1}^{1} Y_{2q} V_q^1 + \sum_{q=2}^{3} Y_{2q} V_q^0 \right] \right]$$

$$= -1 \times Im\left[(V_2^0)^* \left[Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0 \right] \right]$$

$$= -1 \times Im\left[1.03 \left[(-2.035 + j8.609)(1.03) + (4.07 - j17.219) \times (1.03) + (-2.035 + j8.609) \times 1 \right] \right]$$

$$= -1 \times Im\left[0.06288 - 0.26707j \right]$$

$Q_2^1{}_{,cal} = \underline{0.267}$ p.u.

$$V_p^{k+1}{}_{,temp} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_2^1{}_{,temp} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0\right]$$

$$= \frac{1}{4.07 - j17.219}\left[\frac{+1.5 - jQ.267}{1.03} - (-2.035 + j8.609)(1.03) - (-2.035 + j8.609)\times 1\right]$$

$= 1.0481 + j0.0767 \quad = 1.051\angle 4.187^\circ$

$\delta_2^1 = \angle V_2^1{}_{,temp} = \underline{4.187^\circ}$

$V_2^1 = |V_2|_{spec}\angle\delta_2^1 = 1.03\underline{\angle 4.187^\circ}$

$= \underline{1.027 + j0.0752}$ p.u.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - \sum_{q=1}^{2} Y_{3q} V_q^1\right] - \cancel{\sum_{q=3}^{3} Y}$$

$$= \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1\right]$$

$$= \frac{1}{4.07 - j17.219}\left[\frac{-1.2 + j0.5}{1} - (-2.035 + j8.609)\times 1.03 - (-2.035 + j8.609)(1.027 + j0.0752)\right]$$

$= -0.89 - 0.421j$

$= \cancel{0.9308 - 0.2525j} = \cancel{0.9645\angle -15.17^\circ}$

$= 0.9853 - 0.02189j = 0.\underline{9855}\angle -1.272$

17(b) Find the energy stored in the rotor of a three phase, 50 Hz. 250 MVA turbo alternator with H=7.5 MJ/MVA. Determine the value of the inertia constant M. The generator is initially supplying a steady power of 150 MW. If the mechanical power input to the turbine is suddenly decreased to 100 MW, evaluate the initial acceleration of the rotor neglecting all losses. Assume 6 poles. Also find the rotor speed after 10 cycles.

G = m/c rating = 250 MVA

H = 7.5 MJ/MVA . $\frac{7.5 \times 250}{\pi \times 50}$

$M = \frac{GH}{\pi f}$ = 11.94 MJ-s/elect.rad = 0.208 MJ-s/elect.deg

$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M} = \frac{150-100}{11.94} = 4.187$ elect.rad/s²

(poles) $P = 6$. 10 cycles ⇒ 0.2 s.

$\frac{d^2\delta}{dt^2} = \frac{4.187}{P/2} = \frac{4.187}{3} = \cancel{1.396}$ 1.396 mech.rad/s²

$= \frac{60}{2\pi} \times 1.396 = 13.328$ rpm/s

Rotor speed after 10 cycles = 1500 + 13.328 × 0.2 = 1502.66 rpm

3 What are the main functions of load frequency controller in power system?

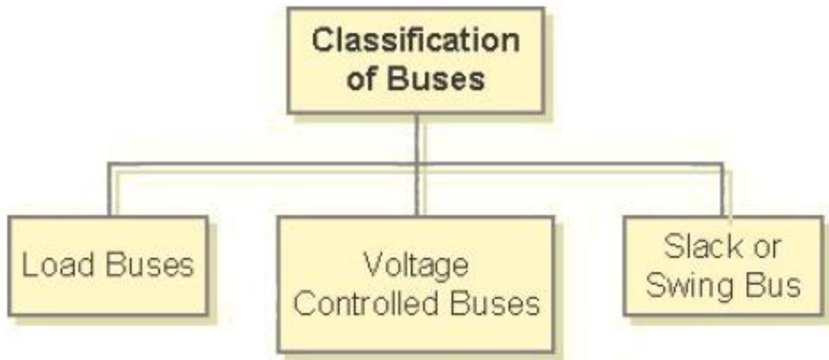
The major purposes of LFC can be summarized as follows:

- Maintaining frequency at transient power loads (unknown external disturbance)
- Regulation of tie line power exchanger error
- Tackling ambiguities in the power system model and the variations

4 Classify the various types of buses in a power system for load flow studies.

Classification Of Buses

For load flow studies it is assumed that the loads are constant and they are defined by their real and reactive power consumption. It is further assumed that the generator terminal voltages are tightly regulated and therefore are constant. The main objective of the load flow is to find the voltage magnitude of each bus and its angle when the powers generated and loads are pre-specified. To facilitate this we classify the different buses of the power system shown in the chart below.



Classification Of Buses

Load Buses : In these buses no generators are connected and hence the generated real power P_{Gi} and reactive power Q_{Gi} are taken as zero. The load drawn by these buses are defined by real power $-P_{Li}$ and reactive power $-Q_{Li}$ in which the negative sign accommodates for the power flowing out of the bus. This is why these buses are sometimes referred to as P-Q bus. The objective of the load flow is to find the bus voltage magnitude $|V_i|$ and its angle δ_i.

Voltage Controlled Buses : These are the buses where generators are connected. Therefore the power generation in such buses is controlled through a prime mover while the terminal voltage is controlled through the generator excitation. Keeping the input power constant through turbine-governor control and keeping the bus voltage constant using automatic voltage regulator, we can specify constant P_{Gi} and $| V_i |$ for these buses. This is why such buses are also referred to as P-V buses. It is to be noted that the reactive power supplied by the generator Q_{Gi} depends on the system configuration and cannot be specified in advance. Furthermore we have to find the unknown angle δ_i of the bus voltage.

Slack or Swing Bus : Usually this bus is numbered 1 for the load flow studies. This bus sets the angular reference for all the other buses. Since it is the angle difference between two voltage sources that dictates the real and reactive power flow between them, the particular angle of the slack bus is not important. However it sets the reference against which angles of all the other bus voltages are measured. For this reason the angle of this bus is usually chosen as 0° . Furthermore it is assumed that the magnitude of the voltage of this bus is known.

Now consider a typical load flow problem in which all the load demands are known. Even if the generation matches the sum total of these demands exactly, the mismatch between generation and load will persist because of the line I^2R losses. Since the I^2R loss of a line depends on the line current which, in turn, depends on the magnitudes and angles of voltages of the two buses connected to the line, it is rather difficult to estimate the loss without calculating the voltages and angles. For this reason a generator bus is usually chosen as the slack bus without specifying its real power. It is assumed that the generator connected to this bus will supply the balance of the real power required and the line losses.

6 What do you mean by penalty factor as referred to economic operation of power system?

Consider a load is distributed between the different plants that are joined by transmission lines, then the line losses have to be explicitly included in the economic dispatch problem. In this section we shall discuss this problem.

When the transmission losses are included in the economic dispatch problem, we can modify (5.4) as

$$P_T = P_1 + P_2 + \cdots + P_N - P_{LOSS}$$

where P_{LOSS} is the total line loss. Since P_T is assumed to be constant, we have

$$0 = dP_1 + dP_2 + \cdots + dP_N - dP_{LOSS}$$

In the above equation dP_{LOSS} includes the power loss due to every generator, i.e.,

$$dP_{LOSS} = \frac{\partial P_{LOSS}}{\partial P_1} dP_1 + \frac{\partial P_{LOSS}}{\partial P_2} dP_2 + \cdots + \frac{\partial P_{LOSS}}{\partial P_N} dP_N$$

Also minimum generation cost implies $df_T = 0$ as given in (5.5). Multiplying both (5.12) and (5.13) by λ and combining we get

$$0 = \left(\lambda \frac{\partial P_{LOSS}}{\partial P_1} - \lambda\right) dP_1 + \left(\lambda \frac{\partial P_{LOSS}}{\partial P_2} - \lambda\right) dP_2 + \cdots + \left(\lambda \frac{\partial P_{LOSS}}{\partial P_N} - \lambda\right) dP_N$$

Adding (5.14) with (5.5) we obtain

$$0 = \sum_{i=1}^{N} \left(\frac{\partial f_T}{\partial P_i} + \lambda \frac{\partial P_{LOSS}}{\partial P_i} - \lambda \right) dP_i$$

The above equation satisfies when

$$\frac{\partial f_T}{\partial P_i} + \lambda \frac{\partial P_{LOSS}}{\partial P_i} - \lambda = 0, \quad i = 1, \ldots, N$$

Again since

$$\frac{\partial f_T}{\partial P_i} = \frac{df_T}{dP_i}, \quad i = 1, \ldots, N$$

from (5.16) we get

$$\lambda = \frac{df_1}{dP_1} L_1 = \frac{df_2}{dP_2} L_2 = \cdots = \frac{df_N}{dP_N} L_N$$

where L_i is called the **penalty factor** of load- i and is given by

$$L_i = \frac{1}{1 - \partial P_{LOSS} / \partial P_i}, \quad i = 1, \ldots, N$$

7 What are the factors affecting transient stability in power system? (5)

Methods to improve transient stability

Factors influencing TS

1. The post fault disturbance system reactance as seen from generator. The weaker the post disturbance system the lower Pmax will be there.
2. The duration of fault clearing time. The longer the fault is applied, the longer the rotor will be accelerated and more KE will be gained. The more energy that is gained during acceleration, the more difficult it is to dissipate it during deceleration

5. The generator loading before the disturbance. The higher the loading, the closure the unit to Pmax will increase, which means during acceleration it is more likely to become unstable.

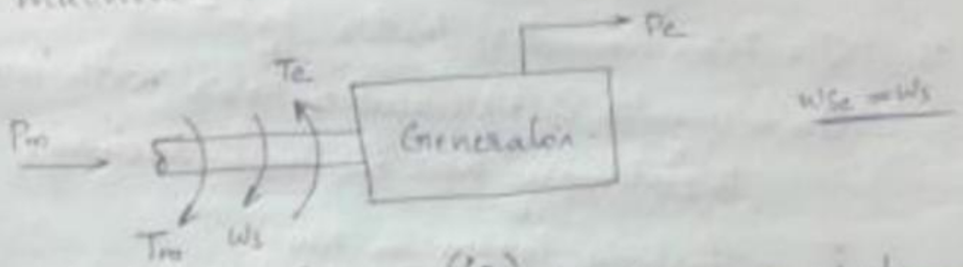
6. The generator internal reactance, the lower the reactance the higher the peak power and the lower the initial rotor angle.

7. The generator o/p during the fault. This is a fn of the fault location and type of fault.

8 What is swing equation? Derive the expression for swing equation for a synch machine connected to an infinite bus.

The Swing Equation

Figure shows the torque speed and flow of mechanical and electrical powers in a synchronous machine



Consider a synchronous M/c (Gn) developing an electromagnetic torque Te (and a corresponding electromagnetic power Pe) while operating at synchronous speed ωe. If the input torque provided by the prime mover at the generator shaft is Tm then under steady state condition (without any disturbance)

$$T_m = T_e$$

The differential equation governing rotor dynamics

$$J \frac{d^2 \theta_m}{dt^2} = T_a = T_m - T_e \quad Nm.$$

angular acceleration.

NB J = Rotor moment of Inertia $kg\,m^2$

θ_m = angular displacement of rotor with respect to stationary axis in mech-radians

T_m = turbine torque in Nm.

T_e = electromagnetic torque ... Nm.

Multiplying both sides with ω_{sm}.

$$J\omega_{sm} \times \frac{d^2\theta_m}{dt^2} \times 10^{-6} = P_m - P_e \quad MW$$

$$\left(J \cdot \left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right) \frac{d^2\theta_e}{dt^2} = P_m - P_e.$$

θ_e = angle in electrical radians.

$$M \frac{d^2\theta_e}{dt^2} = P_m - P_e.$$

It is more convinient to measure the angular position of the rotor w.r.to synchronously rotating frame of referense.

$\delta = \theta_e - \omega_s t$. (torque angle power angle.

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2}$$

$\theta_e = \omega_s t + \delta$

$\frac{d\theta_e}{dt} = \omega_s + \frac{d\delta}{dt}$ — change from synchronous

$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2}$

$$M \frac{d^2\delta}{dt^2} = P_m - P_e \quad MW.$$

$$\frac{GH}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \quad MW.$$

G (MVA rating of M/c)

$$\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad MW.$$

Dividing through out by G (MVA rating of M/c)

$$M_{pu} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e \ (pu)$$

$$M_{pu} = \frac{H}{\pi f}.$$

$$\boxed{\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \ pu}$$ swing equation.

This describes the rotor dynamics of S/M.

9b

Find the expression for three phase power in terms of symmetrical components. (4)

Real and Reactive Power

The three-phase power in the original unbalanced system is given by

$$P_{abc} + jQ_{abc} = V_a I_a^* + V_b I_b^* + V_c I_c^* = V_{abc}^T I_{abc}^*$$

where I^* is the complex conjugate of the vector I. Now from (7.10) and (7.15) we get

$$P_{abc} + jQ_{abc} = V_{a012}^T C^{-T} C^{-1*} I_{a012}^*$$

From (7.11) we get

$$C^{-T} C^{-1*} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore from (7.17) we get

$$P_{abc} + jQ_{abc} = 3\left(V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^*\right)$$

We then find that the complex power is three times the summation of the complex power of the three phase sequences.

16 (a) Explain Equal Area criterion and state the assumptions made. (5)

5.7 EQUAL AREA CRITERION

The transient stability analysis of simple system can be performed by using a simple criterion called equal area criterion.

During the transient state of a power system we may come across the following two situations for changes in δ (torque angle) with respect to time.

i) The δ may increase to a maximum value and then decrease to a stable value. The system is considered as stable.

ii) The δ may keep on increasing indefinitely. In this case the system is unstable.

The above facts can be stated as a stability criterion as given below.

i) The system is stable if, $\frac{d\delta}{dt} = 0$ at some time instant.

ii) The system is unstable if, $\frac{d\delta}{dt} > 0$ for a sufficiently long time (typically 1 second or more)

Consider the swing equation of a generator connected to infinite bus (which is derived in section 5.3, equ 5.29).

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad .....(5.56)$$

Let there be a change in P_e due to a large disturbance, with P_m remaining constant.

Now, $P_m - P_e = P_a$ .....(5.57)

where P_a is the accelerating power.

Also we know that p.u value of M = $H/\pi f$. Hence the equ(5.56) can be written as,

$$M\frac{d^2\delta}{dt^2} = P_a$$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{P_a}{M} \qquad .....(5.58)$$

On multiplying the equ(5.58) by $2\frac{d\delta}{dt}$ we get,

$$2\frac{d\delta}{dt}\ \frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\ \frac{P_a}{M}$$

$$2\frac{d\delta}{dt}\ \frac{d}{dt}\ \frac{d\delta}{dt} = \frac{2}{M}P_a\ \frac{d\delta}{dt}$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}P_a\ \frac{d\delta}{dt}$$

$$\therefore d\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}P_a\ d\delta \qquad .....(5.59)$$

$\frac{d\delta}{dt}$ = Relative speed of M/c with respect to synchronously revolving ref frame.

On integrating the equation (5.59) we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta \qquad .....(5.60)$$

where δ_0 is the initial value of torque angle or rotor angle

On integrating the equation (5.59) we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta \qquad .....(5.60)$$

where δ_0 is the initial value of torque angle or rotor angle

On taking square root of equ(5.60) we get,

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta} \qquad .....(5.61)$$

For a stable system $\frac{d\delta}{dt} = 0$, at a particular time instance. There fore for a stable system the equation (5.61) can be written as

$$\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta} = 0 \qquad .....(5.62)$$

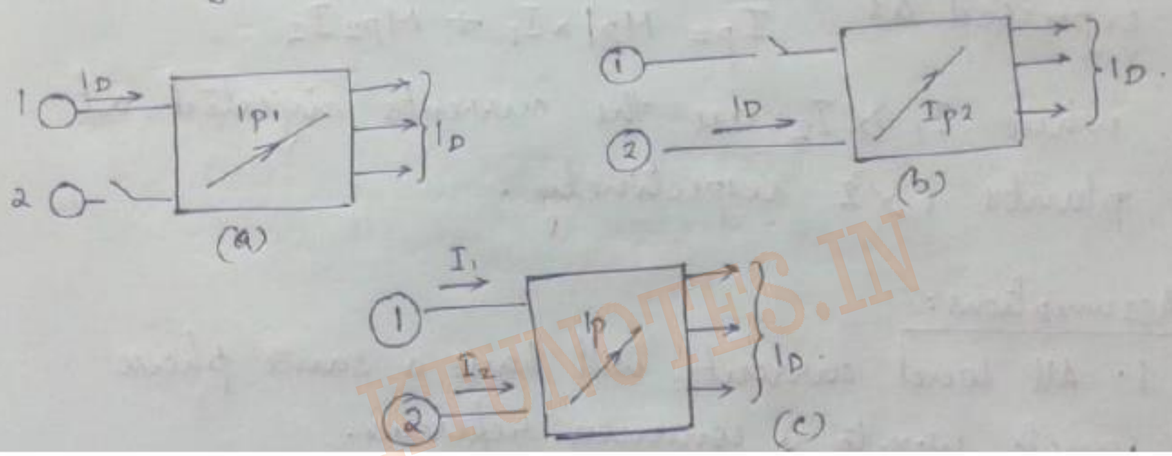
The equation (5.62) is zero if the integral of P_a is zero. Hence we can say that,

$$\text{for } \frac{d\delta}{dt} = 0, \text{ the term } \int_{\delta_0}^{\delta} P_a\, d\delta = 0 \qquad .....(5.63)$$

(b) Derive the expression for transmission losses as a function of power generation.

Derivation of Transmission loss formula

Figure shows the case of two generating plants connected to an arbitary number of loads through a transmission network. One line within the network is designated as branch p



Imagine that the total load current I_D is supplied by plant 1 only, as in Fig 1. Let the current in line p be I_{p1}

$$\text{Define} \quad M_{p1} = \frac{I_{p1}}{I_D}$$

Similarly with plant 2 alone supplying the total load current we can define

$$M_{p2} = \frac{I_{p2}}{I_D}.$$

M_{p1} and M_{p2} are called current distribution factors. Their value depend upon the impedances of the lines and their interconnection and are independent of the current I_D.

When both generator 1 & 2 are supplying current into the N/w as in fig (c) applying the principle of superposition the current in line p can be expressed as. $I_p = M_{p1} \times I_1 + M_{p2} I_2$

where I_1 & I_2 are the currents injected at plants 1 & 2 respectively.

Assumptions:

1. All load currents will have a same phase angle w.r. to a common reference.

Consider the load current at i^{th} bus

$$I_{Di} = \frac{|V_i| \angle \delta_i}{Z_i \angle \phi_i} = |I_{Di}| \angle \delta_i - \phi_i$$
$$= |I_{Di}| \angle \theta_i.$$

2. Ratio $\frac{X}{R}$ is same for all N/w branches.

These two assumptions leads us to the conclusion that I_{p1} & I_D have same phase angle. and so have I_{p2} and I_D. such that the current distribution factors M_{p1} & M_{p2} are real rather than complex.

Let $I_1 = |I_1| \angle \sigma_1$ and $I_2 = |I_2| \angle \sigma_2$

where σ_1 & σ_2 are the phase angles of I_1 & I_2 w.r. to some common reference.

We know $I_p = (M_{p1} I_1 + M_{p2} I_2)^2$

$$|I_p|^2 = (M_{p1} |I_1| \cos \sigma_1 + M_{p2} |I_2| \cos \sigma_2)^2 + (M_{p1} |I_1| \sin \sigma_1 + M_{p2} |I_2| \sin \sigma_2)^2$$

We know $I_p = (M_{p1} I_1 + M_{p2} I_2)$

$$|I_p|^2 = (M_{p1} |I_1| \cos\sigma_1 + M_{p2} |I_2| \cos\sigma_2)^2 + (M_{p1} |I_1| \sin\sigma_1 + M_{p2} |I_2| \sin\sigma_2)^2$$

$$|I_p^2| = M_{p1}^2 |I_1|^2 + M_{p2}^2 |I_2|^2 + 2 M_{p1} M_{p2} |I_1| |I_2| \cos(\sigma_1 - \sigma_2)$$

([illegible])

Now $|I_1| = \dfrac{P_1}{\sqrt{3} V_1 \cos\phi_1}$ & $|I_2| = \dfrac{P_2}{\sqrt{3} V_2 \cos\phi_2}$

where P_1 & P_2 are three phase real power injected at plant 1 & 2

$\cos\phi_1$ & $\cos\phi_2$ are plant 1 pf & plant 2 pf.

V_1 & V_2 are bus voltages at plant 1 & 2

If R_p is the resistance of branch p the total transmission loss is given by.

$$P_L = \sum_{p=1}^{P} 3\,|I_p|^2 R_p$$

$$P_L = \sum_{p=1}^{P} \Bigg[3 \times \Bigg[\frac{M_{p1}^2 \cdot P_1^2 \cdot R_p}{3|V_1|^2 \cos^2\phi_1^2} + \frac{M_{p2}^2 \cdot P_2^2 \cdot R_p}{3|V_2|^2 \cos^2\phi_2^2} + \frac{2 P_1 P_2 \cos(\sigma_1 - \sigma_2)}{3|V_1||V_2| \cos\phi_1 \cos\phi_2} \cdot \sum_p M_{p1} M_{p2} \cdot R_p \Bigg]$$

ie, $P_L = P_1^2 B_{11} + P_2^2 B_{22} + 2 P_1 P_2 B_{12}$.

where $B_{11} = \dfrac{1}{|V_1^2|(\cos^2\phi_1)^2} \sum_p M_{p1}^2 \cdot R_p$.

where $B_{11} = \dfrac{1}{|V_1^2|(\cos^2\phi_1)^2} \sum_p M_{p1}^2 \cdot R_p$.

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2| \cos\phi_1 \cos\phi_2} \sum_p M_{p1} \cdot M_{p2} R_p$$

$$B_{22} = \frac{1}{|V_2^2| \cos\phi_2^2} \cdot \sum_p M_{p2}^2 R_p$$

$$P_L = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{matrix} P_1 B_{11} + P_2 B_{21} \\ P_2 B_{12} + P_2 B_{22} \end{matrix}$$

The terms B_{11}, B_{12}, B_{22} are called loss coefficients or B coefficients.

If voltages are line to line kV, R in Ohms.

unit of B coefficients are MW^{-1}

Further with P_1 & P_2 expressed in MW

P_L will also be in MW.

The above results can be extended to the general case of k plants with transmission loss expressed as

$$P_L = \sum_{m=1}^{k} \sum_{n=1}^{k} P_m B_{mn} P_n.$$

17 (a) What is unit commitment problem? What are the constraints and the solution techniques for unit commitment problem involving thermal plants?

5.2. STATEMENT OF UNIT COMMITMENT PROBLEM

To select the generating units that will supply the forecasted (estimated load in advance) load of the system over a required period of time at minimum cost as well as provide a specified margin of the operating reserve, known as the spinning reserve. This procedure is known as unit commitment.

Tomorrow's unit commitment (UC) problem may be stated as follows :

To determine : If N-generating units, ($2^N - 1$) number of combinations will be obtained. From many feasible subsets, determine the subset of units that would satisfy the expected demand at minimum operating cost.

Loads vary from time to time. So, we are interested not only in determining one subset of units satisfying economically the demand in one particular hour, we want 24 subsets to satisfy the 24 consecutive hour demands per day. This involves consideration of start up and shut down costs as well as constraints on minimum up time/down time of the units.

5.3. CONSTRAINTS IN UNIT COMMITMENT

Each individual power system may impose different rules on the scheduling of units, depends on generation make-up and load curve characteristics, *etc.* The constraints to be considered for unit commitment are:

- Spinning reserve
- Thermal constraints
  - Minimum uptime
  - Minimum downtime
  - Crew Constraint
- Other constraints
  - Hydro constraint
  - Must run constraint
  - Fuel constraint

5.3.1. Spinning Reserve

Spinning reserve is the total amount of generation available from all units synchronized on the system minus the present load and losses being supplied.

$$\text{Spinning reserve} = \begin{Bmatrix} \text{Total amount} \\ \text{of generation} \end{Bmatrix} - [\text{Present load} + \text{Losses}]$$

Spinning reserve must be established, so that the loss of one or more units does not cause drop in system frequency. (*i.e.*, If one unit is lost, the spinning reserve unit has to make up for the loss in a specified time period.

Spinning reserve is the reserve generating capacity running at zero load or no load.

Reserves has

- To avoid transmission system limitations or bottling of reserves.
- To allow some parts of the system to run as islands. (Some area electrically disconnected.)

Reserve Capacity

Capacity in excess of that required to carry peak load.

high power demand occurs or when other generating units are off-line for maintenance, repairs or refueling.

Reserve generating capacity include quick-start diesel or gas turbine unit, or hydro units and pumped-storage hydro-units that can be brought on-line, synchronized and brought up to full capacity quickly.

Automatic generation control system is used to make up for a generation unit failure, and to restore frequency and interchange power through tie-line quickly in the event of generating unit outage.

Reserve Margin

The percentage of installed capacity exceeding the expected peak demand during a specified period.

Typical Rules for Spinning Reserve Set by Regional Reliability Council

- Reserve must be given percentage of forecasted peak demand.
- Reserve must be capable of making up the loss of the most heavily loaded unit in a given period of time.
- Calculate reserve requirements as a function of the probability of not having sufficient generation to meet the load.

5.3.2. Thermal Unit Constraints

A thermal unit can withstand only gradual temperature changes and is required to take some hours to bring the unit on-line. For thermal plants, one hour is the smallest time period that should be considered for unit commitment solution as the start-up and shut-down time for many units is of this order.

The thermal unit constraints are minimum up time, minimum down time and crew constraints.

Minimum up time

Once the unit is running, it should not be turned off immediately.

Minimum down time

Once the unit is decommitted, there is a minimum time before it can be recommitted.

Crew constraints

If a plant consists of two or more units, they cannot both be turned on at the same time. Since there are not enough crew members to attend both units while starting up.

Start-up cost

It is dependent upon the down-time of the unit *i.e.*, the time interval between shut-down and restart.

$$\text{Start-up cost} = 0, \text{ if unit is stopped and started immediately.}$$

(a) Start-up cost when cooling

During down time period, the unit's boiler to cool down and then heat back up to operating temperature in time for a scheduled turn on.

$$\text{Start-up cost} \propto \text{Cooling of the unit}$$

$$\text{Start-up cost when cooling} = C_C\left(1 - e^{\frac{-t}{\alpha}}\right) \times F + C_f \quad \ldots (5.1)$$

where C_c = Cold start cost

F = Fuel cost

C_f = Fixed cost (includes crew expenses, maintenance expenses)

α = Thermal time constant for the unit

t = Time in which the unit was cooled

5.3.3.2. Must Run Constraints

Some units like nuclear units are given a must-run status during certain times of the year to maintain the voltage in the transmission system.

5.3.3.3. Fuel Constraints

If thermal and hydro sources are available, a combined operation is economic and advantageous. *i.e.*, to minimize the fuel cost of thermal unit over a commitment period.

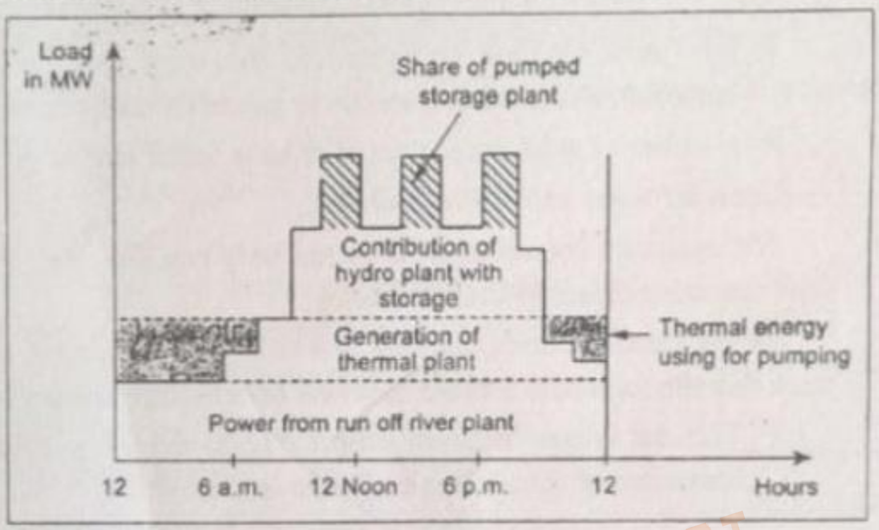


Fig. 5.2. Daily load curve

A typical daily load curve is as shown in Fig.5.2. Power from run off river plant is always operating *i.e.*, base loads. During certain period, thermal power plant is working to meet out the load and to pump water in the back bay. In the next block, hydro plant with storage will operate, then pumped storage plants come into operation to meet out the load during peak load.

A system in which some units have limited fuel, or else have constraints that require them to burn a specified amount of fuel in a given time, presents a most challenging unit commitment problem.

12 Explain the computational procedure for load flow solution using fast decoupled load flow method.

2.11 Fast-decoupled load-flow (FDLF) technique

An important and useful property of power system is that the change in real power is primarily governed by the charges in the voltage angles, but not in voltage magnitudes. On the other hand, the charges in the reactive power are primarily influenced by the charges in voltage magnitudes, but not in the voltage angles. To see this, let us note the following facts:

(a) Under normal steady state operation, the voltage magnitudes are all nearly equal to 1.0.

(b) As the transmission lines are mostly reactive, the conductances are quite small as compared to the susceptance ($G_{ij} << B_{ij}$).

(c) Under normal steady state operation the angular differences among the bus voltages are quite small ($\theta_i - \theta_j \approx 0$ (within $5^o - 10^o$)).

(d) The injected reactive power at any bus is always much less than the reactive power consumed by the elements connected to this bus when these elements are shorted to the ground ($Q_i << B_{ii}V_i^2$).

With these facts at hand, let us re-visit the equations for Jacobian elements in Newton-Raphson (polar) method (equation (2.48) to (2.55)). From equations (2.50) and (2.51) we have,

$$\begin{aligned}
\frac{\partial P_i}{\partial V_j} &= 2V_iG_{ii} + \sum_{\substack{k=1\\ \neq i}}^{n} V_kY_{ik}\cos(\theta_i - \theta_k - \alpha_{ik}) \\
&= 2V_iG_{ii} + \sum_{\substack{k=1\\ \neq i}}^{n} V_kY_{ik}\left[\cos(\theta_i - \theta_k)\cos\alpha_{ik} + \sin(\theta_i - \theta_k)\sin\alpha_{ik}\right] \\
&= 2V_iG_{ii} + \sum_{\substack{k=1\\ \neq i}}^{n} V_k\left[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right]; \quad j = i \qquad (2.74)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P_i}{\partial V_j} &= V_iY_{ij}\cos(\theta_i - \theta_j - \alpha_{ij}) \\
&= V_iY_{ij}\left[\cos(\theta_i - \theta_j)\cos\alpha_{ij} + \sin(\theta_i - \theta_j)\sin\alpha_{ij}\right] \\
&= V_i\left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right]; \quad j \neq i \qquad (2.75)
\end{aligned}$$

Now, G_{ii} and G_{ij} are quite small and negligible and also $\cos(\theta_i - \theta_j) \approx 1$ and $\sin(\theta_i - \theta_j) \approx 0$, as $[(\theta_i - \theta_j) \approx 0]$. Hence,

$$\frac{\partial P_i}{\partial V_i} \approx 0 \quad \text{and} \quad \frac{\partial P_i}{\partial V_j} \approx 0 \quad \Longrightarrow \mathbf{J_2} \approx 0 \qquad (2.76)$$

Similarly, from equations (2.52) and (2.53) we get,

$$\frac{\partial Q_i}{\partial \theta_j} = \sum_{\substack{k=1\\ \neq i}}^{n} V_iV_k\left[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right]; \quad j = i \qquad (2.77)$$

$$\frac{\partial Q_i}{\partial \theta_j} = -V_iV_j\left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right]; \quad j \neq i \qquad (2.78)$$

Again in light of the natures of the quantities G_{ii}, G_{ij} and $(\theta_i - \theta_j)$ as discussed above,

$$\frac{\partial Q_i}{\partial \theta_i} \approx 0 \quad \text{and} \quad \frac{\partial Q_i}{\partial \theta_j} \approx 0 \quad \Longrightarrow \mathbf{J_3} \approx 0 \tag{2.79}$$

Substituting equations (2.76) and (2.79) into equation (2.40) one can get,

$$\boxed{\begin{bmatrix} \Delta \mathrm{P} \\ \Delta \mathrm{Q} \end{bmatrix} = \begin{bmatrix} \mathrm{J}_1 & 0 \\ 0 & \mathrm{J}_4 \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}} \tag{2.80}$$

In other words, $\mathbf{\Delta P}$ depends only on $\boldsymbol{\Delta\theta}$ and $\mathbf{\Delta Q}$ depends only on $\boldsymbol{\Delta V}$. Thus, there is a decoupling between '$\mathbf{\Delta P}$ - $\boldsymbol{\Delta\theta}$' and '$\mathbf{\Delta Q}$ - $\boldsymbol{\Delta V}$' relations. Now, from equations (2.48) and (2.49) we get,

$$\begin{aligned} \frac{\partial P_i}{\partial \theta_j} &= -\sum_{\substack{k=1 \\ \neq i}}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ &= V_i V_i Y_{ii} \sin(\theta_i - \theta_i - \alpha_{ii}) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ &= -B_{ii} V_i^2 - Q_i \approx -B_{ii} V_i^2; \quad j = i \quad [\text{as} \quad Q_i << B_{ii} V_i^2] \end{aligned} \tag{2.81}$$

$$\begin{aligned} \frac{\partial P_i}{\partial \theta_j} &= V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}); \quad j \neq i \\ &= V_i V_j Y_{ij} \left[\sin(\theta_i - \theta_j) \cos \alpha_{ij} - \cos(\theta_i - \theta_j) \sin \alpha_{ij}\right]; \quad j \neq i \\ &= V_i V_j \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\right]; \quad j \neq i \\ &= -V_i V_j B_{ij}; \quad j \neq i \end{aligned} \tag{2.82}$$

Similarly, from equations (2.54) and (2.55) we get,

$$\begin{aligned} \frac{\partial Q_i}{\partial V_j} &= -2V_i B_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} V_i &= -2V_i^2 B_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} V_i &= -V_i^2 B_{ii} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = Q_i - V_i^2 B_{ii}; \quad j = i \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} V_i &= -V_i^2 B_{ii}; \quad j = i \quad [\text{as} \quad Q_i << B_{ii} V_i^2] \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} &= -V_i B_{ii}; \quad j = i \end{aligned} \tag{2.83}$$

$$\begin{aligned}\frac{\partial Q_i}{\partial V_j} &= V_i Y_{ij}\sin(\theta_i-\theta_j-\alpha_{ij}); \quad j\neq i\\ &= V_i Y_{ij}\left[\sin(\theta_i-\theta_j)\cos\alpha_{ij}-\cos(\theta_i-\theta_j)\sin\alpha_{ij}\right]; \quad j\neq i\\ &= V_i\left[G_{ij}\sin(\theta_i-\theta_j)-B_{ij}\cos(\theta_i-\theta_j)\right]; \quad j\neq i\\ &\approx -V_i B_{ij}; \quad j\neq i\end{aligned} \tag{2.84}$$

Combining equations (2.80)-(2.82) we get, $\Delta P_i = -V_i\sum_{k=1}^{n} V_k B_{ik}\Delta\theta_k$. Or,

$$\frac{\Delta P_i}{V_i} = -\sum_{k=1}^{n} V_k B_{ik}\Delta\theta_k \tag{2.85}$$

Now, as $V_i \approx 1.0$ under normal steady state operating condition, equation (2.85) reduces to,

$$\frac{\Delta P_i}{V_i} = -\sum_{k=1}^{n} B_{ik}\Delta\theta_k. \quad \text{Or,}\ \frac{\mathbf{\Delta P}}{\mathbf{V}} = [-\mathrm{B}]\,\boldsymbol{\Delta\theta}. \quad \text{Or,}$$

$$\boxed{\frac{\mathbf{\Delta P}}{\mathbf{V}} = [B']\,\boldsymbol{\Delta\theta}} \tag{2.86}$$

Matrix B' is a constant matrix having a dimension of $(n-1)\times(n-1)$. Its elements are the negative of the imaginary part of the element (i,k) of the $\mathbf{Y_{BUS}}$ matrix where $i = 2,3,\cdots\cdots n$ and $k = 2,3,\cdots\cdots n$.

Again combining equations (2.80), (2.83) and (2.84) we get,

$$\Delta Q_i = -V_i\sum_{k=1}^{n} B_{ik}\Delta V_k. \quad \text{Or,}\ \frac{\Delta Q_i}{V_i} = -\sum_{k=1}^{n} B_{ik}\Delta V_k. \quad \text{Or,}$$

$$\boxed{\frac{\mathbf{\Delta Q}}{\mathbf{V}} = [B'']\,\mathbf{\Delta V}} \tag{2.87}$$

Again, $[B'']$ is also a constant matrix having a dimension of $(n-m)\times(n-m)$. Its elements are the negative of the imaginary part of the element (i,k) of the $\mathbf{Y_{BUS}}$ matrix where $i = (m+1),(m+2),\cdots\cdots n$ and $k = (m+1),(m+2),\cdots\cdots n$. As the matrixes $[B']$ and $[B'']$ are constant, it is not necessary to invert these matrices in each iteration. Rather, the inverse of these matrices can be stored and used in every iteration, thereby making the algorithm faster. Further simplification in the FDLF algorithm can be made by,

a. Ignoring the series resistances is calculating the elements of $[B']$. Also, by omitting the elements of $[B']$ that predominantly affect reactive power flows, i.e., shunt reactances and transformer off nominal in phase taps.

b. Omitting from $[B'']$ the angle shifting effect of phase shifter, which predominantly affects real power flow.

In the next lecture, we will look at an example of FDLF method.