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	APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY SIXTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), MAY 2019	
	Course Code: EE306	
	Course Name: POWER SYSTEM ANALYSIS	
Max. M	Tarks: 100Duration: 3	Hours
	PART A Answer all questions, each carries5 marks.	Marks
1	Define the term per unit quantity. Enumerate Merits and Demerits of P.U	(5)
2	What is the significance of current limiting reactors in power system? Where are they located? Give examples.	(5)
3	How slack bus differs from other buses in a power system? What is the	(5)
	significance of slack bus in load flow analysis?	
4	What is AVR? What are the functions?	(5)
5	Derive condition for economic load dispatch neglecting losses.	(5)
6	Define penalty factors and loss coefficients in economic operation of power system.	(5)
7	Explain the terms 1) steady state stability 2) dynamic stability 3) transient stability	(5)
8	Write all methods to improve steady state stability limit of power system	(5)
	PART B Answer any two full questions, each carries10 marks.	
9	A 300 MVA, 20kV three phase generator has a subtransient reactance of 20%. The generator supplies two synchronous motors over a 64km transmission line having transformers at both ends as shown on the single line diagram. The ratings of the motors are:M1-200MVA, 13.2kV, X"=20%; M2- 100MVA, 13.2kV, X"=20%. The ratings of transformers are T1-350MVA, 230/20 kV, X=10%; T2- composed of 3 single phase transformers each rated 127/ 13.2kV,100MVA, X=10%. Series reactance of the transmission line is 0.5	(10)

Page 1 of 3

(10)

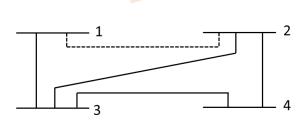
- 10 a) Draw the zero sequence networks of star-delta and delta-delta transformers (5)
 - b) Draw and explain the oscillogram of short circuit current when an unloaded (5) alternator is subjected to a 3-phase fault
- 11 Derive the expression for fault current and draw the interconnection of sequence (10) networks for the following faults on the terminals of an unloaded generator.
 - (a) single Line to Ground fault
 - (b) Line to Line fault

PART C Answer any two full questions, each carries10 marks.

12

The figure shows the SLD of a simple four bus system. The table gives the line impedance identified by the buses on which these terminate. The shunt admittance at all the buses is assumed to be negligible.

- a) Find Y_{BUS} , assuming that the line shown dotted is not connected.
- b) What modifications need to be carried out in Y_{BUS} if the line shown dotted is connected



Line, Bus to Bus	R pu	X pu
1-2	0.05	0.15
1-3	0.10	0.30
2-3	0.15	0.45
2-4	0.10	0.30
3-4	0.05	0.15

- 13 a) Compare between Gauss-Seidal method and Newton-Raphson method, in load (5) flow studies.
 - b) With neat diagram explain the working of a turbine speed governing system. (5)
- 14 Derive the generator load model and draw the complete block diagram of a single (10) area system



PART D

Answer any two full questions, each carries 10 marks.

Assume that the fuel input Btu/hr for units 1 and 2 are given by

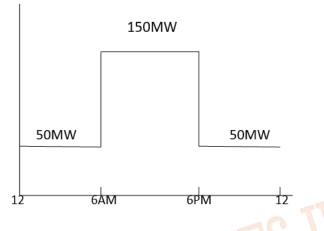
(10)

(5)

$$F_1 = (8P_1 + 0.024P_1^2 + 80)10^6$$

$$F_2 = (6P_2 + 0.04P_2^2 + 120)10^6$$

The maximum and minimum loads on the units are 100MW and 10MW respectively. Determine the minimum cost of generation when the following load is supplied. The cost of fuel is Rs²/million Btu.



- 16 a) What is the significance of spinning reserve constraint in unit commitment (5) problem? Explain with example.
 - b) Explain the equal area criterion to determine the stability of a power system (5)
- 17 a) Derive the swing equation.
 - b) A 2 pole 50 Hz, 11kV turbo generator has a rating of 60 MW at 0.85 p.f lagging. (5) Its rotor has a moment of inertia of 8800 kg-m². Calculate its inertia constant in MJ/MVA and its angular momentum in MJ-s/elect. Degree.

15

1. Define the term per unit quantity. Enumerate Merits and Demerits of P.U.

Ans:-

A per-unit system is the expression of system quantities as fractions of a defined base unit quantity.

$$Per Unit Value = \frac{Actual Value in any unit}{Press on use for any unit}$$

Base or reference value in the same unit

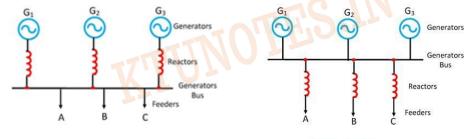
Voltages, currents and impedances expressed in per unit do not change when they are referred from one side of transformer to the other side. This is a great advantage.

- Per unit impedances of electrical equipment of similar type usually lie within a narrow range, when the equipment ratings are used as base values.
- > Transformer connections do not affect the per unit values.
- Manufacturers usually specify the impedances of machines and transformers in per unit or percent of name plate ratings.
- 2. What is the significance of current limiting reactors in power system? Where are they located? Give examples.

Ans:-

The current limiting reactor is an inductive coil having a large inductive reactance's in comparison to their resistance and is used for limiting short circuit currents during fault conditions. Current-voltage reactors also reduce the voltage disturbances on the rest of the system. The main purpose of the current limiting reactor is that its reactance should not decrease when a large short current flows through its windings.

It is installed in feeders and ties, in generators leads, and between bus sections, for reducing the magnitude of short circuit currents and the effect of the respective voltage disturbance.



Generator Reactors

Feeders Reactors

3. How slack bus differs from other buses in a power system? What is the significance of slack bus in load flow analysis?

Ans:-

Slack Bus/Reference Bus/Swing Bus:- Slack bus in a power system absorb or emit the active or reactive power from the power system. The slack bus does not carry any load.

In electrical power systems a slack bus (or swing bus) is used to balance the active power |P| and reactive power |Q| in a system while performing load flow studies.

The slack bus is used to provide for system losses by emitting or absorbing active and/or reactive power to and from the system. It is difficult to estimate the loss without calculating the voltages and angles. For this reason a generator bus is usually chosen as the slack bus without specifying its real power. It is assumed that the generator connected to this bus will supply the balance of the real power required and the line losses.

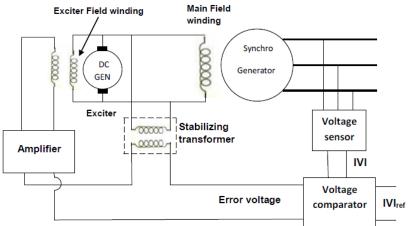
This bus sets the angular reference for all the other buses. Since it is the angle difference between two voltage sources that dictates the real and reactive power flow between them, the particular angle of the slack bus is not important.

4. What is AVR? What are the functions?

Ans:-

An automatic voltage regulator (AVR) is an electronic device for automatically maintaining generator output terminal voltage at a set value under varying load and operating temperature. It compares the rectified output of a voltage transformer, proportional to the terminal voltage of the generator, (compounding transformer-to step down the voltage) with the reference voltage and generates an error signal equal to their difference and give it to the exciter field.

Automatic voltage regulator on a generator stops fluctuation of voltage produced by it during various load conditions.



5. Derive condition for economic load dispatch neglecting losses. Ans:-

The problem is to find the real power generation for each plant such that the objective function (i.e., total production cost) as defined by the equation

$$C_t = \sum_{i=1}^{n_g} C_i$$
$$= \sum_{i=1}^n \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

Is minimum, subjected to the constraints,

$$\sum_{i=1}^{n_g} P_i = P_D$$

A typical approach is to augment the constraints into objective function by using the Lagrange multipliers

$$\mathcal{L} = C_t + \lambda \left(P_D - \sum_{i=1}^{n_g} P_i \right)$$

The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

First condition, given by $\frac{\partial \mathcal{L}}{\partial P_i} = 0$ results in

$$\frac{\partial C_t}{\partial P_i} + \lambda(0-1) = 0$$

Since

$$C_t = C_1 + C_2 + \dots + C_{n_g}$$

then

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \ \lambda$$

and therefore the condition for optimum dispatch is

$$rac{dC_i}{dP_i} = \lambda \qquad i = 1, \dots, n_g$$

or

$$\beta_i + 2\gamma_i P_i = \lambda$$

When losses are neglected with no generator limits, for most economic operation. all plants must operate at equal incremental production cost

6. Define penalty factors and loss coefficients in economic operation of power system.

Ans:-

 $\frac{\partial P_L}{\partial P_n}$ is known as the incremental transmission loss at plant n and $\left(\frac{1}{1-\frac{\partial P_L}{\partial P_n}}\right)$ is called as the penalty

factor. A penalty factor is employed to add a penalty to the generation cost in the cost minimization function of that plant which has a transmission loss on delivering to a load.

The B_{mn} coefficients are the loss coefficients and for an n generator system the coefficient is an n × n symmetric matrix

They are real loss coefficients. They are assumed to be constant and reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficients. They are usually represented as Bmn.

$$B_{mn} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix}$$

The diagonal elements are all positive and strong as compared with the off diagonal elements which mostly are negative and are relatively weaker.

7. Explain the terms 1) steady state stability 2) dynamic stability 3) transient stability.

Ans:-

Steady-state Stability:-

Steady-state stability relates to the response of synchronous machine to a gradually increasing load. It is basically concerned with the determination of the upper limit of machine loading without losing synchronism, provided the loading is increased gradually. **Dynamic Stability:-**

Dynamic stability involves the response to small disturbances that occur on the system, producing oscillations. The system is said to be dynamically stable if theses oscillations do not acquire more than certain amplitude and die out quickly. If these oscillations continuously grow in amplitude, the system is dynamically unstable. The source of this type of instability is usually an interconnection between control systems.

Transient Stability:-

Transient stability involves the response to large disturbances, which may cause rather large changes in rotor speeds, power angles and power transfers. Transient stability is a fast phenomenon usually evident within a few second.

8. Write all methods to improve steady state stability limit of power system.

Ans:-

By reducing the X (reactance) or by raising the |E| or by increasing the |V|, the improvement of steady state stability limit of the system is possible.

Reduction of transfer reactance

A power system which has a lower value of transfer reactance can have better steady-state stability limit. This can be achieved by:

i) use of parallel lines

If the power has to be transferred through long distance transmission lines, use of parallel lines reduce transfer reactance as well as improve voltage regulations.

ii) use of series capacitors

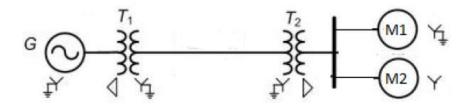
Similarly series capacitors are sometimes employed in lines to get the same features.

- Increase in the magnitudes of E and V.
- Higher and fast field excitation system enhances steady-state power limits
- Two systems to improve the stability limit are quick excitation voltage and higher excitation voltage.

PART B

Answer any two full questions, each carries10 marks.

9. A 300 MVA, 20kV three phase generator has a subtransient reactance of 20%. The generator supplies two synchronous motors over a 64km transmission line having transformers at both ends as shown on the single line diagram. The ratings of the motors are:M1-200MVA, 13.2kV, X"=20%; M2- 100MVA, 13.2kV, X"=20%. The ratings of transformers are T1-350MVA, 230/20 kV, X=10%; T2- composed of 3 single phase transformers each rated 127/ 13.2kV,100MVA, X=10%. Series reactance of the transmission line is 0.5 ohm/km. Draw the reactance diagram with all reactance's marked in p.u. Select the generator ratings as base values.



Ans:-

Base MVA = 300

Base voltage at generator side = 20 kV

Base voltage in transmission line = 230 kV

Line to line voltages of transformer T2 : $\sqrt{3} \times 127 / 13.2 = 220 / 13.2 \text{ kV}$

Base voltage at motor side = 230 x 13.2/220 = 13.8 kV

Per-unit reactance of generator = 0.2

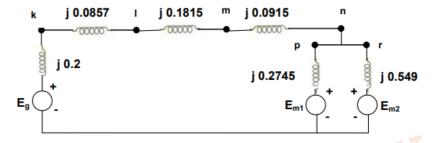
Per-unit reactance of transformer T1 = 0.1 x 300/350= 0.0857

Per-unit reactance of transmission line = $0.5 \times 64 \times 300/230^2 = 0.1825$

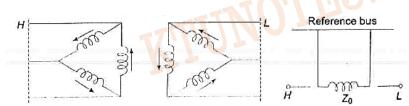
Per-unit reactance of transformer T2 = $0.1 \times (220/230)^2 = 0.0915$

Per-unit reactance of motor M1 = 0.2 x (300/200) x (13.2/13.8)²=0.2745

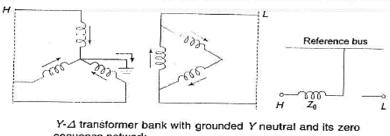
Per-unit reactance of motor M1 = $0.2 \times (300/100) \times (13.2/13.8)^2 = 0.549$



10.a)Draw the zero sequence networks of star-delta and delta-delta transformers Ans:-



 Δ - Δ transformer bank and its zero sequence network.



sequence network

b) Draw and explain the oscillogram of short circuit current when an unloaded alternator is subjected to a 3-phase fault.

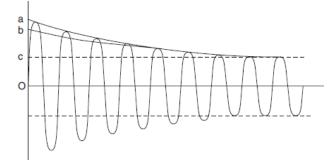
Ans:-

Whenever a 3-phase short circuit occurs at the terminals of an alternator, the current in the armature circuit increases suddenly to a large value and since the resistance of the circuit then is small as compared to its reactance, the current is highly lagging and the p.f. is approximately zero.

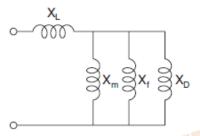
Due to this sudden switching, as analysed in the previous section, there are two components of currents:

(i) a.c. component,

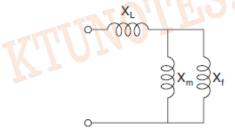
(ii) d.c. component (decaying).



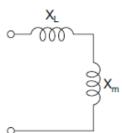
At the instant of the short-circuit there is mutual coupling between the stator winding, rotor winding and the damper winding and equivalent circuit is shown in Fig



The effect of damper winding and the eddy current in the pole faces disappears after the first few cycles. Accordingly, the equivalent circuit after first few cycles reduces to the one shown in Fig.



After a few more cycles depending upon the time constant of the field winding the effect of the d.c. component will die down and steady state conditions will prevail.



The reactance in the initial stage is called the sub transient reactance; corresponding to 2nd Figure is called as the transient reactance and the steady state reactance is the synchronous reactance i3rd equivalent circuit. It can be seen from the equivalent circuit that the inductance increases as from the initial stage to the final steady state i.e.,

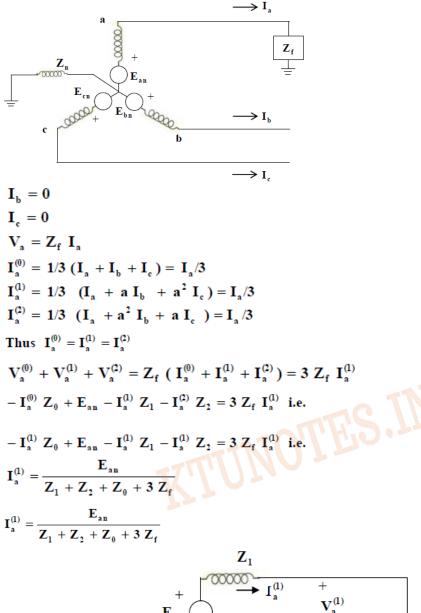
synchronous reactance > transient reactance > subtransient reactance.

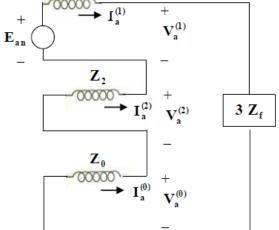
11.Derive the expression for fault current and draw the interconnection of sequence networks for the following faults on the terminals of an unloaded generator.

(a) single Line to Ground fault

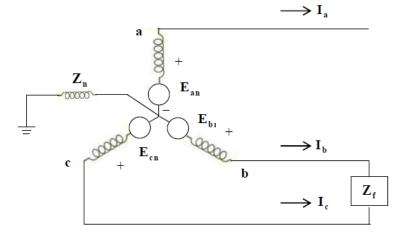
Ans:-

The circuit diagram is shown in Fig





(b) Line to Line fault Ans:-

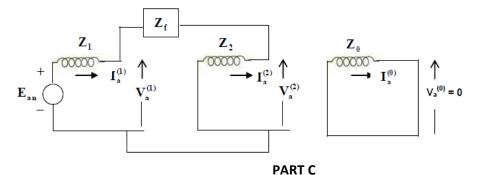


The fault conditions are $I_{a} = 0$ $I_{\rm h} + I_{\rm c} = 0$ $\mathbf{V}_{\mathbf{h}} - \mathbf{Z}_{\mathbf{f}} \mathbf{I}_{\mathbf{h}} = \mathbf{V}_{\mathbf{h}}$ $I_{2} = 0$ $\mathbf{I}_{\rm h} + \mathbf{I}_{\rm c} = \mathbf{0}$ $V_b - Z_f I_b = V_c$ Then $I_a^{(0)} = 1/3 (I_a + I_b + I_c) = 0$ $I_a = 1/3 (I_a + 1_b + 1_c)^{-2}$ $I_a^{(1)} = 1/3 (I_a + 1_b + 1_c)^{-2} = I_b/3 (1_a - 1_c)^{-2}$ $I_a^{(2)} = 1/3 (I_a + a^2 I_b + a I_c) = I_b/3 (a^2 - a)$ Since $I_a^{(0)} = 0$, $V_a^{(0)} = -Z_0 I_a^{(0)} = 0$ Further $I_a^{(2)} = -I_a^{(1)}$ From eqn. (27) $V_{a}^{(0)} + a^2 V_{a}^{(1)} + a V_{a}^{(2)} - Z_{f} (a^2 I_{a}^{(1)} + a I_{a}^{(2)}) = V_{a}^{(0)} + a V_{a}^{(1)} + a^2 V_{a}^{(2)}$ $(a^2 \neq a) V_a^{(1)} - Z_f (a^2 \neq a) I_a^{(1)} = (a^2 - a) V_a^{(2)}$ Thus $V_a^{(1)} - Z_f I_a^{(1)} = V_a^{(2)}$ From the above eqn. $E_{an} - Z_1 I_a^{(1)} - Z_f I_a^{(2)} = -Z_2 I_a^{(2)} = Z_2 I_a^{(1)}$ $\mathbf{E}_{an} = (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_f) \mathbf{I}_a^{(1)}$ i.e. $I_a^{(1)} = \frac{E_{an}}{Z_1 + Z_2 + Z_f}$ Therefore Z_f

Therefore

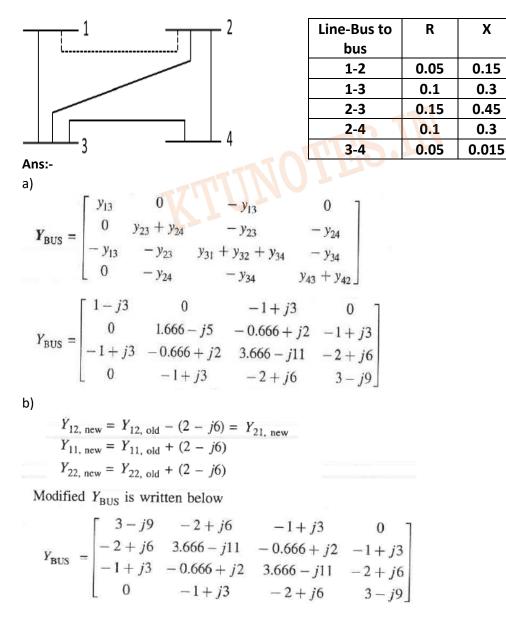
$$I_{a}^{(1)} = \frac{E_{an}}{Z_{1} + Z_{2} +}$$
$$I_{a}^{(2)} = -I_{a}^{(1)}$$

and $I_a^{(0)} = 0; V_a^{(0)} = 0$



Answer any two full questions, each carries 10 marks.

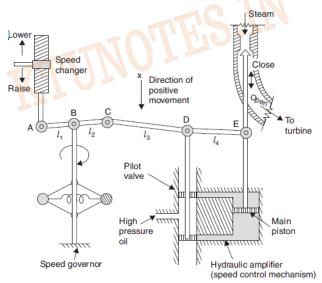
- 12. The figure shows the SLD of a simple four bus system. The table gives the line impedance identified by the buses on which these terminate. The shunt admittance at all the buses is assumed to be negligible.
- a) Find YBUS, assuming that the line shown dotted is not connected.
- b) What modifications need to be carried out in YBUS if the line shown dotted is connected?



13.a) Compare between Gauss-Seidal method and Newton-Raphson method, in load flow studies. Ans:-

S.No	G.S	N.R	
1	Require large number of iterations to reach convergence	Require less number of iterations to reach convergence. Computation time per iteration is more	
2	Computation time per iteration is less		
3	It has linear convergence characteristics	It has quadratic convergence characteristics	
4	The number of iterations required for convergence increases with size of the system	The number of iterations are independent of the size of the system	
5	Less memory requirements	More memory requirements.	

b) With neat diagram explain the working of a turbine speed governing system. Ans:-



The real power control mechanism of a generator is shown in Figure.

The main parts are: 1) Speed changer 2) Speed governor 3) Hydraulic amplifier 4) Control valve. They are connected by linkage mechanism. Their incremental movements are in vertical direction. The movements are assumed positive in the directions of arrows. Corresponding to "raise" command, linkage movements will be: "A" moves downwards; "C" moves upwards; "D" moves upwards; "E" moves downwards. This allows more steam or water flow into the turbine resulting incremental increase in generator output power.

When the speed drops, linkage point "B" moves upwards and again generator output power will increase.

14.Derive the generator load model and draw the complete block diagram of a single area system Ans:-

The increment in power input to the generator-load system is

$$\Delta P_G - \Delta P_D$$

This increment in power input to the system is accounted for in two ways:

(i) Rate of increase of stored kinetic energy in the generator rotor. At scheduled frequency (f^{0}), the stored energy is:

$$W_{ke}^0 = H \times P_r$$

The kinetic energy at a frequency of $(f^0 + \Delta f)$ is given by:

$$W_{\lambda e} = W_{\lambda e}^{0} \left(\frac{f^{0} + \Delta f}{f^{0}} \right)^{2}$$
$$\Box HP_{r} \left(1 + \frac{2\Delta f}{f^{0}} \right)$$

Rate of change of kinetic energy:

$$\frac{d}{dt}\left(W_{ks}\right) = \frac{2HP_r}{f^0}\frac{d}{dt}\left(\Delta f\right)$$

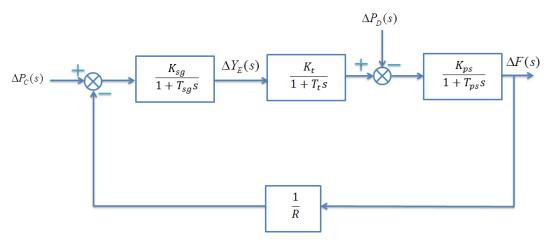
(ii) The rate of change of load with respect to frequency:

$$\left(\frac{\partial P_D}{\partial f}\right) \times \Delta f = B \Delta f$$

Power balance equation:

$$\begin{split} \Delta P_{G} - \Delta P_{D} &= \frac{2HP_{r}}{f^{0}} \frac{d}{dt} (\Delta f) + B\Delta f \\ \Delta P_{G}(pu) - \Delta P_{D}(pu) &= \frac{2H}{f^{0}} \frac{d}{dt} (\Delta f) + B(pu)\Delta f \\ \Delta F(s) &= \frac{\Delta P_{G}(s) - \Delta P_{D}(s)}{B + \frac{2H}{f^{0}} s} \\ &= \Delta P_{G}(s) - \Delta P_{D}(s) \left(\frac{K_{ps}}{1 + T_{ps} s}\right) \\ \Delta F(s) &= \Delta P_{G}(s) - \Delta P_{D}(s) \left(\frac{K_{ps}}{1 + T_{ps} s}\right) \\ \Delta P_{G}(s) & \longrightarrow \\ \Delta P_{G}(s) & \longrightarrow \\ \Delta P_{G}(s) & \longrightarrow \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) - \Delta F(s) \\ \Delta F(s) &= \Delta F(s) \\ \Delta F(s) \\ \Delta F(s) &= \Delta F(s) \\ \Delta$$

Block Diagram model of Load Frequency Control



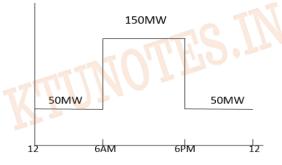
PART D Answer any two full questions, each carries 10 marks.

15.Assume that the fuel input Btu/hr for units 1 and 2 are given by

$$F1 = (8P_1 + 0.024P_1^2 + 80)10^6$$

$$F2 = (6P_2 + 0.04P_2^2 + 120)10^6$$

The maximum and minimum loads on the units are 100MW and 10MW respectively. Determine the minimum cost of generation when the following load is supplied. The cost of fuel is Rs'2/million Btu.



Ans:-

From the fuel input characteristics

$$\frac{dF_1}{dP_1} = 0.048 P_1 + 8$$
$$\frac{dF_2}{dP_2} = 0.08 P_2 + 6$$

(i) When load is 50 MW, for economic loading the conditions are

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$\begin{aligned} P_1 + P_2 &= 50 \\ 0.048 \ P_1 + 8 &= 0.08 \ P_2 + 6 \\ P_1 + P_2 &= 50 \end{aligned}$$

From these equations,

$$\begin{split} P_1 &= 15.625 \text{ MW and } P_2 = 34.375 \text{ MW} \\ F_1 &= 210.868 \text{ million Btu per hr.} \\ F_2 &= 373.5 \text{ million Btu per hr.} \end{split}$$

(ii) When load is 150 MW, the equations are

 $0.048 P_1 + 8 = 0.08P_2 + 6$ $P_1 + P_2 = 150$

Solving

 $P_1=78.126~{\rm MW}~~{\rm and}~~P_2=71.874~{\rm MW}$ $F_1=851.496$ million Btu/hr. $F_2=757.87$ million Btu/hr.

Total cost = Rs. (210.868 + 373.5 + 851.496 + 757.87) × 12 × 2

= Rs. 52649.61

16.a) What is the significance of spinning reserve constraint in unit commitment problem? Explain with example.

Ans:-

Spinning Reserve is the total amount of generation available from all units synchronized on the system minus the present load and losses being supplied

SR=Total amount of generation- (Present load + Losses)

Loss of one unit should not cause drop in frequency. Spinning Reserve of active capacity is capacity reserve located at operating units and units with the start-up time of up to 5 minutes. Thus, a fast-start reserve is also a Spinning Reserve. Used to meet an unexpected increase in demand and to ensure power supply in the event of any generating unit suffering a forced outage.

b) Explain the equal area criterion to determine the stability of a power system. Ans:-

Thus the principle by which stability under transient conditions is determined without solving the swing equation, but makes use of areas in power angle diagram, is called the EQUAL AREA CRITERION.

The swing equation for the alternator connected to the infinite bus bars is

$$M \frac{d^2 \delta}{dt^2} = P_s - P_e$$

Multiplying both sides by $d\delta/dt$, we get

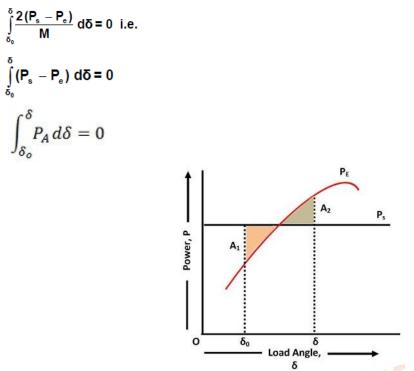
$$M \frac{d^2 \delta}{dt^2} \frac{d \delta}{dt} = (P_s - P_e) \frac{d \delta}{dt} \qquad i.e. \qquad \frac{1}{2} M \frac{d}{dt} (\frac{d \delta}{dt})^2 = (P_s - P_e) \frac{d \delta}{dt}$$

Thus

$$\frac{d}{dt} \left(\frac{d\delta}{dt}\right)^2 \frac{dt}{d\delta} = \frac{2(P_s - P_e)}{M} ; \qquad \text{i.e.} \quad \frac{d}{d\delta} \left(\frac{d\delta}{dt}\right)^2 = \frac{2(P_s - P_e)}{M}$$
On integration
$$\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta} \frac{2(P_s - P_e) d\delta}{M} \qquad \text{i.e.} \quad \frac{d\delta}{dt} = \sqrt{\int_{\delta_0}^{\delta} \frac{2(P_s - P_e) d\delta}{M}}$$

Before the disturbance occurs, δ_0 was the torque angle. At that time $d\delta/dt = 0$. As soon as the disturbance occurs, $d\delta/dt$ is no longer zero and δ starts changing.

Torque angle δ will cease to change and the machine will again be operating at synchronous speed after a disturbance, when $d\delta/dt = 0$ or when



The area A_1 represents the kinetic energy stored by the rotor during acceleration, and the A_2 represents the kinetic energy given up by the rotor to the system, and when it is all given up, the machine has returned to its original speed.

The area under the curve P_A should be zero, which is possible only when P_A has both accelerating and decelerating powers, i.e., for a part of the curve $P_S > P_E$ and for the other $P_E > P_S$. For a generation action, $P_S > P_E$ for the positive area and A1> P_S for negative areas A_2 for stable operation. Hence the name equal area criterion.

17.a) Derive the swing equation.

Ans:-

If Ts represents the shaft torque and Te the electromagnetic torque and if these are assumed positive for a generator, the net torque causing acceleration is

Ta = Ts – Te

and Ta is positive if shaft torque input is greater than the electromagnetic power output. For a motor if Te the electromagnetic torque input is greater than the shaft torque output the motor rotor will accelerate. A similar relation holds good when expressed in terms of power,

i.e.,

Pa = Ps – Pe

where Pa is accelerating power. Since a synchronous machine is a rotating body, the laws of mechanics apply to this also. We know that power is equal to torque times the angular velocity.

 $Pa = Ta\omega$

Now torque is moment of inertia times the angular acceleration.

 \therefore Pa = Ta ω = I $\alpha\omega$ = M α

Where M = I ω

Here ω is the angular velocity in mechanical radians per sec, i.e., M is in joule-sec/mechanical radian.

The acceleration α can be expressed in terms of the angular position of the rotor as

The angle θ changes continuously with respect to time when a sudden change occurs in the system. The value of θ is given by

Taking the derivative of equation

$$\frac{d\theta}{dt} = \omega_r + \frac{d\delta}{dt}$$
$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$$
$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_c$$

This is the swing equation

b) A 2 pole 50 Hz, 11kV turbo generator has a rating of 60 MW at 0.85 p.f lagging. Its rotor has a moment of inertia of 8800 kg-m². Calculate its inertia constant in MJ/MVA and its angular momentum in MJ-s/elect. Degree Ans:-

$$G = \frac{P}{pf} = \frac{60}{0.85} = 70.5MVA$$

$$N = \frac{120f}{P} = \frac{120 \times 50}{2} = 3000 \ rpm$$

$$KE = \frac{1}{2}J\omega^{2} = \frac{1}{2}J\left(\frac{2\pi N}{60}\right)^{2}$$

$$= \frac{1}{2}8800\left(\frac{2\pi 3000}{60}\right)^{2} = 434.262MJ$$

$$H = \frac{Kinetic \ Energy}{Machine \ MVA \ rating} = \frac{434.262}{70.5} = 6.152$$

$$M = \frac{GH}{180\ f} = \frac{70.5 \times 6.152}{180 \times 50} = 0.04819\ \text{MJ} - \text{s/elect. Degree}$$